EE 102A - Assignment 1

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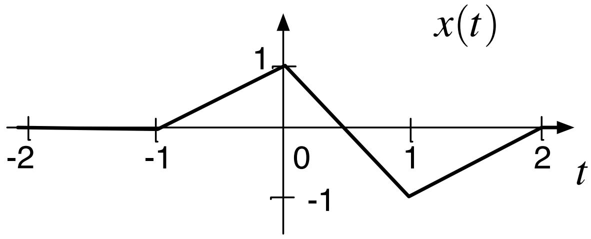
Problem 1. Let x be defined as shown in the figure below.



Draw the following signals:
a) $x\left(2\left(-t+\frac{1}{2}\right)\right)$
b) $x\left(\frac{t-1}{2}\right)$

Problem 2. *Step Function and Rect*
The unit step function u is defined as

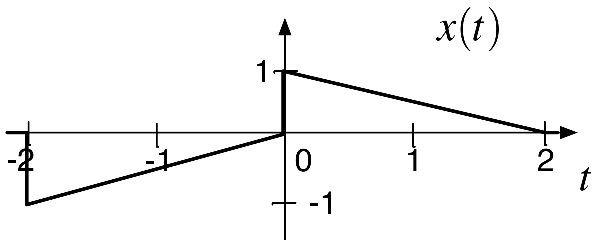
$$u(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}.$$

The rect function Π is defined as

$$\Pi(x) = \begin{cases} 1 & |x| < 1/2 \\ 0.5 & |x| = 1/2 \\ 0 & |x| > 1/2 \end{cases}.$$

Express Π as a simple combination of (modified) unit step functions.

Problem 3. *Even and Odd*
a) Show that any real function f can be written as a sum of an even function f^{even} and an odd function f^{odd}.
b) Find the even and odd decomposition of the function x from the first problem.
c) Find the even and odd decomposition of this signal:



Problem 4.
a) Express $\cos(\omega t)$ as a simple function of complex exponentials.
b) Express $\sin(\omega t)$ as a simple function of complex exponentials.

Problem 5. Write a Matlab function that accepts three (x, y) points and returns the area of a triangle. The prototype of the function should be as follows:

```
function area = areaOfTriangle( x1, y1, x2, y2, x3, y3 )
  % relevant code goes here
```

Provide your code and show two test cases of your code working.
Hint: there's a property of cross product that's very relevant here.

Problem 6. Calculate the derivative and antiderivative of each of the following expressions:
a) *(Just the derivative)* $f(x) = \int_{-\infty}^{x} \exp(-i\,8\gamma)d\gamma$
b) $f(x) = 18\exp(-i\,2\pi\,x)$
c) $f(x) = \exp(-i8x)\cos(2\pi x)$
d) $f(x) = i\cos(2\pi x)\sin(3\pi x)$

Problem 7. Find $\int_{-\infty}^{\infty} e^{-\pi x^2} dx$.

Problem 8. The variable x is a complex number.
a) How many unique square roots of x exist?
(Note: if $x = k\angle\,\theta$ and $y = k\angle\,\theta + 2\pi$ then $x = y$.)
b) Find all the fifth roots of 1.
c) Find all the fifth roots of $1 + 1i$.

Problem 9. *Periodic Functions*
a) Let $x : \mathbb{R} \to \mathbb{C}$ be an odd periodic function with fundamental period T. What is the value of $x(3T)$?
b) Let $x_1, x_2 : \mathbb{R} \to \mathbb{C}$ be two periodic functions with periods T_1 and T_2, respectively. What relationship must T_1 and T_2 satisfy so that $x_1 + x_2$ is also periodic? What is the period of $x_1 + x_2$ if that relationship is satisfied?
c) Find a function where any real number serves as a period of that function. What can you say about any such function?

Problem 10. Suppose $x_1 = a_1 + i\,b_1$ and $x_2 = a_2 + i\,b_2$ are complex numbers.
a) Derive an algebraic expression for $x_1\,x_2$ in terms of a_1, b_1, a_2, and b_2 based on the definition of complex multiplication provided in class.
b) Given that division is the inverse of multiplication, derive an algebraic expression for x_1/x_2 in terms of a_1, b_1, a_2, and b_2.

Problem 11.
a) Prove that for any real number a, $a \cdot 0 = 0$ (where $\cdot$ represents multiplication).
b) Either prove or disprove that for any complex number a, $a \cdot 0 = 0$.

Problem 12. Either prove or disprove the statement "The function $h(f) = \exp(i\,2\,\pi\,f)$ is a periodic function." If it is periodic, what is the fundamental period of h?

Problem 13. Evaluate the following integral (where $m, n \in \mathbb{Z}$)

$$\frac{1}{P}\int_{p_0}^{p_0+P} \exp\left(i2\pi\frac{nx}{P}\right)\exp\left(-i2\pi\frac{mx}{P}\right)dx.$$

Problem 14. The energy of a function x is

$$E_x = \int_{-\infty}^{\infty} |x(\gamma)|^2 d\gamma.$$

A function x is an energy function means E_x is defined and non-zero.
The power of a function x is

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(\gamma)|^2 d\gamma.$$

A function x is a power function means P_x is defined and non-zero. Note that the power P_x is the average energy.
Determine whether the following functions are energy and/or power functions.
a) $x(t) = e^{-|t|}$
b) $x(t) = \frac{1}{\sqrt{t}} u(t-1)$
c) $x(t) = e^{-|t|}\cos(2\pi t)$
d) $x(t) = e^{t}\ u(-t)$

Note that $u : \mathbb{R} \to \mathbb{R}$ is the *step function* defined as follows:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases} .$$

Problem 15. *Energy and Power*
a) What is the Power of an Energy function?
b) What is the Energy of a Power function?