# EE 102A - Assignment 2

Nicholas Dwork

**Problem 1.** You are provided with three samples of your function: $(0,2), (1,0)$, and $(2,5)$.
a) Interpolate linearly to estimate the values at $0.1$ and $1.6$.
b) Interpolate parabolically to estimate the values at $0.1$ and $1.6$.

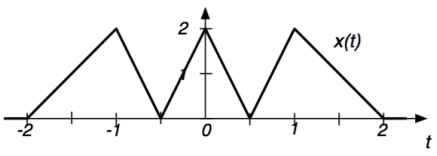
**Problem 2.** Is dot product linear in the first argument? In the second argument?

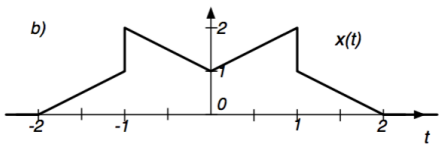
**Problem 3.** A function $y : \mathbb{R} \to \mathbb{R}$ is periodic with fundamental period $T$. The function $y$ is also equal to the sum of two other functions: $y = x_1 + x_2$. Must $x_1$ and $x_2$ be periodic? Either prove that they must be or find a counter-example.

**Problem 4.** Let $x : \mathbb{R} \to \mathbb{R}$ such that $x(\gamma) = (1/\gamma)u(\gamma - 1)$. Is $x$ an Energy function? Is $x$ a Power function?

**Problem 5.** *System Properties*
Determine whether each of the following systems is linear or nonlinear, shift invariant or shift variant, has memory or is memoryless, and is causal or non-causal.
a) $S\{x\}(\gamma) = x(\gamma)\, \sin(\omega\gamma + \phi)$
b) $S\{x\}(t) = t\, x'(t)$
c) $S\{x\}(\gamma) = 1 + x(\gamma)\, \cos(\omega\, \gamma)$
d) $S\{y\}(t) = \cos(\omega\, t + y(t))$
e) $S\{y\}(x) = \int_{-x}^{x} y(\tau)\, d\tau$
f) $S\{z\}(\gamma) = \int_{-\infty}^{\gamma/2} z(\tau)\, d\tau$
g) $S\{y(t+3)\}(t) = y(t+2) + y(t-3)$

**Problem 6.** Write the following signals as a combination (sums and/or products) of rectangle $\Pi$ and triangle $\Lambda$ functions.




**Problem 7.** Let $S$ be the system defined as follows:

$$y(t) = S\{x\}(t) = \frac{x^2(t)}{\int_{-\infty}^{t} x(\tau)\, d\tau}.$$

Is this system linear? Why or why not?

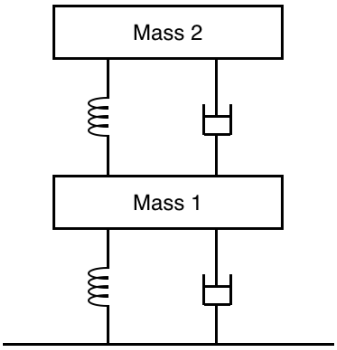
**Problem 8.** Let $G$ be the system that accepts a function $x$ and outputs the even part of the function $x^e$. That is, $G\{x\} = x^e$. Is this system linear or non-linear? Is this system shift invariant

or shift variant?

**Problem 9.** A linear shift invariant system $S$ is applied to a periodic function $x$ with fundamental period $T$. Either prove that the output is periodic or provide an example where the output isn't periodic. If it is periodic, can you determine the period of the output?

**Problem 10.** Let $x, y \in \mathbb{C}^N$. Show that $x \cdot y = \overline{y \cdot x}$.

**Problem 11.** A vehicle (mass 1) is connected to its tires through a spring and a damper. The car is carrying a load (mass 2); the load is connected to the car through a different spring and a different damper as shown in the figure below.




The vehicle is traveling to the right at a known speed. If we consider the altitude of the ground as a function of time $x(t)$ as the input and the altitude of the load as a function of time $y_2(t)$ as the output, then this is a system. Is this system linear? Is this system causal? Is this system shift invariant? Justify each of your answers.

**Problem 12.** (Review) Find all solutions to the following differential equation

$$y''(x) - 36\,y'(x) + 224\,y(x) = 0,$$

where $y(0) = 0$.