Introduction to Computer Graphics and Computer Vision

Assignment 2: Due 7/23/2015

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1 Main Puzzles

P 1 Visit https://studio.code.org/flappy/1 and complete the puzzles.

P 2 Rewrite the following system of equations using matrices and vectors, then find the solution using Matlab.

$$-3 = 2w - x + 5y + z$$
$$-32 = 3w + 2x + 2y - 6z$$
$$-40 = w + 3x + 3y - z + 7$$
$$49 = 5w - 2x - 3y + 3z$$

P 3 (*Credit: Dan Meyer*) View the video located at the following address. What are the blacked out percentages?

`www.stanford.edu/~ndwork/si2015/hmwk2/act1.mov`

P 4 Below is an image of a box; it has a height of h, a width of w, and a length of l. Find the length of the red diagonal. (Explain your answer using the pythagorean theorem.) How does this length relate to the L_2 norm?
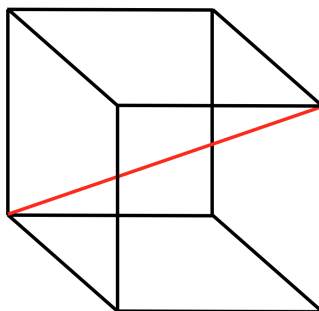
P 5 Let x, y be three element column vectors. Show $x^T y = y^T x$.

P 6   Download the image at `www.stanford.edu/~ndwork/si2015/imgs/bac.jpg`. For this problem, convert this image to grayscale using `rgb2gray`. Note: even if you turn in the rest of your homework handwritten, please turn in the answer to this puzzle electronically (to save paper and ink).

Part a   Make an image $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \frac{1}{50}, \frac{1}{100}$ the size of the original image by saving every 2nd, 3rd, 4th, 5th, 10th, 20th, 50th, 100th pixel and display these images. (Write this as a loop so that you can automatically create each image every time you run the loop. Add `close all` to the beginning of your program to close any open figures, perhaps from a previous run. Use `figure` to create a new figure for each image.) Can you say anything about the distortion of the image (how does it appear to your eye)?

Part b   Now do the same thing, except instead of taking every N^{th} pixel, use the average of blocks of $N \times N$ pixels. So in the case where you are reducing the image size by $\frac{1}{2}$, for the first value take the average of the first 2×2 block, and for the second value compute the average of the second 2×2 block, etc. How does the distortion of these images compare to the distortion of the images from Part a?

Part c   Use Matlab's publish feature to publish this code and its results to a pdf document.

P 7   Show that the two methods of multiplying matrices by a vector discussed in class are equivalent.

P 8   Here we will discuss the following matrix

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Part a   Is M invertible with respect to Matrix-vector multiplication? Either prove that it is or give a counter example.

Note: If you're familiar with determinants, you may be tempted to use determinants here. But we haven't introduced those in class. So if you use any facts associated with determinants, you must first define a determinant and prove any fact that you use. There's a much easier way to solve this puzzle.

Part b   So far we have discussed inversion with respect to Matrix-vector multiplication. What about addition and subtraction? Is M invertible with respect to Matrix-Matrix addition? Is it invertible with respect to Matrix-Matrix subtraction? Either prove that it is or give a counter example for each case.

Part c   What are all the possible values of x that satisfy the following equation?

$$Mx = \begin{bmatrix} 8 \\ 0 \end{bmatrix}$$

part d   What are all the possible values of x that satisfy the following equation?

$$\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix} x = \begin{bmatrix} \pi \\ 2\pi \end{bmatrix}$$

P 9 (*Credit: Dan Meyer*)   View the video of the man falling and determine his speed right before he impacts the airbag.

    `www.stanford.edu/~ndwork/si2015/hmwk2/actone.mov`

P 10 (*Credit: Dr. Boyd*) In this puzzle we consider several linear functions of a grayscale image with $N \times N$ pixels. To keep the matrices small enough to work out by hand, we will consider the case when $N = 3$ (which would hardly qualify as an image). We represent a 3×3 image as a vector in $\mathbb{R}^9$ using the ordering of pixels shown below. (This is called *column-major* ordering.)

1	4	7
2	5	8
3	6	9

Each of the operations or transformations below defines a function $y = f(x)$, where x is the original 9-vector, and y is the 9-vector that corresponds to the transformed image. For each of the following operations, give the 9×9 matrix A for which $y = Ax$.

    Part a   Turn the original image upside down.

    Part b   Rotate the original image clockwise by $\frac{\pi}{2}$ radians.

    Part c   Translate the image up by 1 pixel and right by 1 pixel. In the translated image, assign the value $y_i = 0$ to the pixels in the first column and the last row.

    Part d   Set each pixel y_i to be the average of the neighbors of pixel i in the original image. By neighbors, we mean the pixels immediately above and below, and immediately to the left and right. The center pixel has 4 neighbors, corner pixels have two neighbors, and the remaining pixels have 3 neighbors.

P 11   The Triangle Inequality states that for any two vectors $a, b \in \mathbb{R}^n$,

$$\|a + b\|_2 \leq \|a\|_2 + \|b\|_2.$$

    Part a   Give a geometric interpretation of this expression with pictures drawn in $\mathbb{R}^2$.

    Part b   Prove the Triangle Inequality algebraically.

    Part c   Using the Triangle Inequality, show that the following (called the *Reverse Triangle Inequality*) is true:
$$\|a + b\|_2 \geq \|a\|_2 - \|b\|_2.$$

P 12   The image of a function is the set of all possible outputs of that function. The image of a matrix M is the set of all possible values of Mv where v is any vector. The image of a matrix is directly related to its columns; explain how they're related.

P 13 Suppose we have a line defined by the vector $v = (1, 2, 1)$ and a point $a = (0, 0, 0)$. Suppose we have a plane defined by the normal vector $n = (-1, -1, 0)$ and a point $a_p = (8, 5)$. Does the line intersect the plane? If so, where?

2 Challenge Puzzles

CP 1 Repeat the puzzle where you shrank the image of the dog eating ice cream, but this time do it in color. (Again, if you turn in the rest of your homework handwritten, please turn in this problem electronically.)

CP 2 The Cauchy-Schwartz inequality states that for any two vectors $u, v \in \mathbb{R}^n$,

$$|u \cdot v| \leq \|u\|_2 \|v\|_2.$$

Prove the Cauchy-Schwartz inequality.

CP 3 (*Credit: Dr. Boyd*) Suppose $x \in \mathbb{R}^{100}$ such that $\|x\|_2 = 1$.

    Part a What is the maximum number of entries that can satisfy $|x_i| \geq 3$.

    Part b What are all of the possible values of the mean of x?

CP 4 LATEX is a very useful markup language for typing up math homework. Open an account at www.sharelatex.com, find an online tutorial[1], and write up as much of this homework in LATEX as you can. (Note, this is extremely low priority; please only do this if you've completed the rest of this assignment well.)

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[1]e.g. http://www.latex-tutorial.com