Digital Signals

Assignment 1: Due 6/23/2016

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1 Main Problems

Problem 1 Download the following files from the course website.

- www.stanford.edu/~ndwork/si2016/hmwk1/helloWorld.py
- www.stanford.edu/~ndwork/si2016/hmwk2/showImage.py
- www.stanford.edu/~ndwork/si2016/stanford.jpg

Run hellowWorld and showImage in PyCharm.

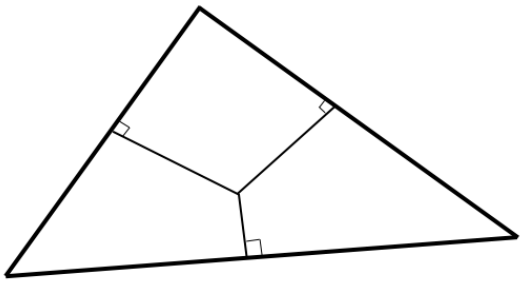
Problem 2 Show in three dimensions that the length of a vector is $a = (a_1, a_2, a_3)$ is $\sqrt{a_1^2 + a_2^2 + a_3^2}$. Draw a picture as part of your explanation.

Problem 3 If you're new to programming, try completing this Hour of Code:

`https://studio.code.org/hoc/1`

This will let you become familiar with programming concepts like "if", "while", and "for".

Problem 4 Show that the perpendicular bisectors of a triangle are concurrent (as shown in the figure below). Your answer should be short and elegant. Try to show it with in as few lines as possible.



Problem 5 Rewrite the following system of equations using matrices and vectors, then find the solution using Python.

$$\begin{aligned} -3 &= 2w - x + 5y + z \\ -32 &= 3w + 2x + 2y - 6z \\ -40 &= w + 3x + 3y - z + 7 \\ 49 &= 5w - 2x - 3y + 3z \end{aligned}$$

Problem 6 Write a function that accepts three (x, y) points and returns the area of a triangle. The prototype of the function should be as follows:

```
def areaOfTriangle( x1, y1, x2, y2, x3, y3 )
  # relevant code goes here
  return area
```

Hint: there's a property of cross product that's very relevant here.

Problem 7 (*Credit: Dan Meyer*) For this problem you will need to view videos on the class website.

Part a At the following website, you will see a video of a jug getting filled with water. How long will it take to fill up the jug? (What information do you think you need? You can ask the TAs for any measurement. If you need something but it's not clear how you would measure it, you must tell the TAs how you would measure it.)

`www.stanford.edu/~ndwork/si2016/hmwk1/act1.mov`

Once you have an answer, present it to a TA. He will show you another video. Why is your answer different from the video?

Part b At the following website, you will see a video of a jug getting getting emptied. How long will it take the jug to empty? (Again, ask the TAs for the information you need.)

`www.stanford.edu/~ndwork/si2016/hmwk1/act1-sequel.mov`

Once you have an answer, present it to a TA. He will show you another video. Why is your answer different from the video?

Problem 8 (*Credit: Dr. Boyd*) A block vector is a vector of vectors. Let x be a block vector with two vector elements,

$$x = \begin{bmatrix} a \\ b \end{bmatrix},$$

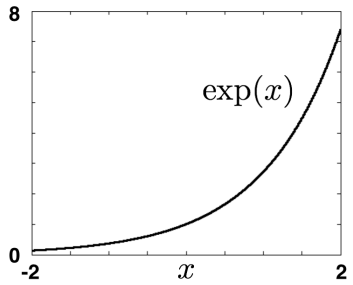
where a and b are vectors of size m and n, respectively.

Part a Express $\text{mean}(x)$ in terms of $\text{mean}(a)$ and $\text{mean}(b)$.

Part b Express $\|x\|_2$ in terms of $\|a\|_2$ and $\|b\|_2$.

Problem 9 It's easy to tell whether or not a function is invertible on a finite domain given its graph. How can you tell?

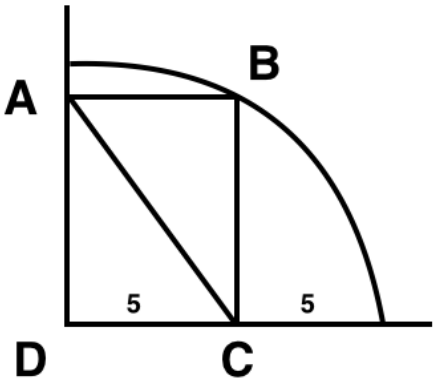
Problem 10 Graph the inverse function of exp for all the points shown on the following graph.



Problem 11 Consider a matrix A. The inverse of A is a matrix A^{-1} such that $A\,A^{-1} = I$ and $A^{-1}\,A = I$. Find the inverse of the following matrix without using a computer.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Problem 12 Find the length of line AC. (Hint, this should take you almost no time at all). *Source: My Best Mathematical and Logic Puzzles by Martin Gardner.*



Problem 13 *Properties of the cross product.*

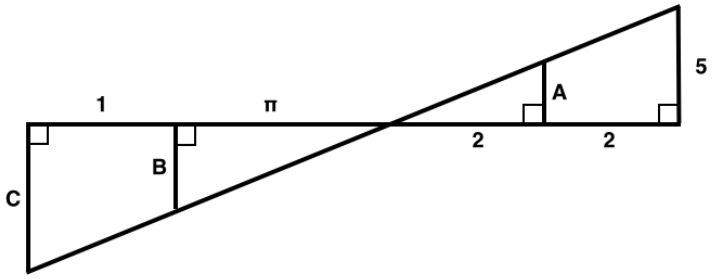
Part a Prove that $a \times b$ is perpendicular to both a and b.

Part b Prove that the cross product of parallel vectors is 0.

Part c How is $a \times b$ related to $b \times a$?

Part d Given a vector a, find a matrix M_a such that $M_a b = a \times b$ for all vectors b.

Problem 14 Find the lengths of lines $A, B,$ and C in the image below.



Problem 15 DeMorgan's Law states

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}.$$

Prove these statements.

2 Challenge Problems

CP 1 Make a function that accepts a natural number as input and outputs a boolean. The function must determine whether or not the input problem is prime; it returns True if the number is prime and False otherwise. The function prototype is as follows.

```
def isPrime( N )
  # relevant code goes here
  return trueOrFalse
```

CP 2 (*Credit: Dr. Boyd*) Suppose $x \in \mathbb{R}^{100}$ such that $\|x\|_2 = 1$.

Part a What is the maximum number of entries that can satisfy $|x_i| \geq 3$.

Part b What are all of the possible values of the mean of x?

CP 3 The triangle inequality states that for two vectors a and b,

$$\|a + b\|_2 \leq \|a\|_2 + \|b\|_2.$$

Prove this.