

# Digital Signals

Assignment 1: Due 7/16/2016 at 21:00

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## 1 Main Problems

**Problem 0** Download and install Steam from <http://store.steampowered.com/>.

**Problem 1** Download the following files from the course website.

- [www.nicholasdwork.com/teaching/si2016/session2/hmwk1/helloWorld.py](http://www.nicholasdwork.com/teaching/si2016/session2/hmwk1/helloWorld.py)
- [www.nicholasdwork.com/teaching/si2016/session2/hmwk1/showImage.py](http://www.nicholasdwork.com/teaching/si2016/session2/hmwk1/showImage.py)
- [www.nicholasdwork.com/teaching/si2016/session2/hmwk1/stanford.jpg](http://www.nicholasdwork.com/teaching/si2016/session2/hmwk1/stanford.jpg)

Run helloWorld and showImage in PyCharm.

**Problem 2** Show in three dimensions that the length of a vector is  $a = (a_1, a_2, a_3)$  is  $\sqrt{a_1^2 + a_2^2 + a_3^2}$ . Draw a picture as part of your explanation.

*Note: understanding this concept will be key to succeeding in this course. Please make sure you understand the solution to this problem well.*

**Problem 3** If you're new to programming, try completing this Hour of Code: <https://studio.code.org/hoc/1> This will let you become familiar with programming concepts like “if”, “while”, and “for”.

**Problem 4** Rewrite the following system of equations using matrices and vectors, then find the solution using Python.

$$\begin{aligned} -3 &= 2w - x + 5y + z \\ -32 &= 3w + 2x + 2y - 6z \\ -40 &= w + 3x + 3y - z + 7 \\ 49 &= 5w - 2x - 3y + 3z \end{aligned}$$

**Problem 5** (*Credit: Dan Meyer*) For this problem you will need to view videos on the class website.

**Part a** At the following website, you will see a video of a jug getting filled with water. How long will it take to fill up the jug? (In this problem we are simulating a scientific inquiry. We don't want to wait all the time it would take to fill the jug, but we want to know how long it would take. What information do you think you need? How would you go about finding that information? You can ask the TAs for any measurement. If you need something but it's not clear how you would measure it, you must tell the TAs how you would measure it.) [www.nicholasdwork.com/teaching/si2016/videos/act1.mov](http://www.nicholasdwork.com/teaching/si2016/videos/act1.mov) Once you have an answer, present it to a TA. He will then show you another video. Why is your answer different from the video?

**Part b** At the following website, you will see a video of a jug getting getting emptied. How long will it take the jug to empty? (Again, ask the TAs for the information you need.) [www.nicholasdwork.com/teaching/si2016/videos/act1-sequel.mov](http://www.nicholasdwork.com/teaching/si2016/videos/act1-sequel.mov) Once you have an answer, present it to a TA. He will show you another video. Why is your answer different from the video?

**Problem 6** *Properties of the cross product.*

**Part a** Prove that  $a \times b$  is perpendicular to both  $a$  and  $b$ .

**Part b** Prove that the cross product of parallel vectors is 0.

**Part c** How is  $a \times b$  related to  $b \times a$ ?

**Problem 7** (*Credit: Dr. Boyd*) A block vector is a vector of vectors. Let  $x$  be a block vector with two vector elements,

$$x = \begin{bmatrix} a \\ b \end{bmatrix},$$

where  $a$  and  $b$  are vectors of size  $m$  and  $n$ , respectively.

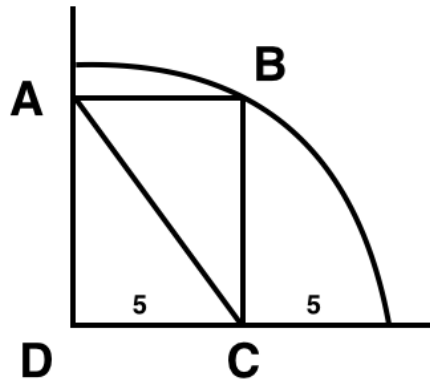
**Part a** Express  $\text{mean}(x)$  in terms of  $\text{mean}(a)$  and  $\text{mean}(b)$ .

**Part b** Express  $\|x\|_2$  in terms of  $\|a\|_2$  and  $\|b\|_2$ .

**Problem 8** Consider a matrix  $A$ . The inverse of  $A$  is a matrix  $A^{-1}$  such that  $AA^{-1} = I$  and  $A^{-1}A = I$ . Find the inverse of the following matrix without using a computer. (Note, you are not permitted to use the determinant in your solution; we will not discuss or use determinants at all in this class.)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

**Problem 9** Find the length of line AC. (Hint, this should take you almost no time at all).  
*Source: My Best Mathematical and Logic Puzzles by Martin Gardner.*



**Problem 10** Write a function that accepts three  $(x, y)$  points and returns the area of a triangle. The prototype of the function should be as follows:

```
def areaOfTriangle( x1, y1, x2, y2, x3, y3 )
    # relevant code goes here
    return area
```

*Hint: there's a property of cross product that's very relevant here.*

**Problem 11** DeMorgan's Law states

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

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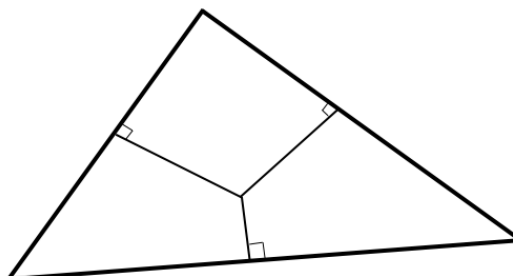
Prove these statements.

**Problem 12** The triangle inequality states that for two vectors  $a$  and  $b$ ,

$$\|a + b\|_2 \leq \|a\|_2 + \|b\|_2.$$

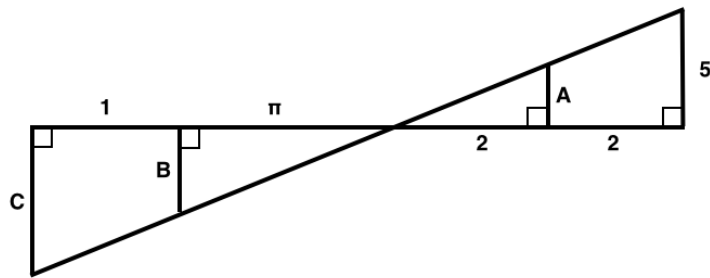
Prove this.

**Problem 13** Show that the perpendicular bisectors of a triangle are concurrent (as shown in the figure below). Your answer should be short and elegant. Try to show it with in as few lines as possible.



## 2 Challenge Problems

**CP 1** Find the lengths of lines  $A$ ,  $B$ , and  $C$  in the image below.



**CP 2** Make a function that accepts a natural number as input and outputs a boolean. The function must determine whether or not the input problem is prime; it returns True if the number is prime and False otherwise. The function prototype is as follows.

```
def isPrime( N )
    # relevant code goes here
    return trueOrFalse
```

**CP 3** (*Credit: Dr. Boyd*) Suppose  $x \in \mathbb{R}^{100}$  such that  $\|x\|_2 = 1$ .

**Part a** What is the maximum number of entries that can satisfy  $|x_i| \geq 3$ .

**Part b** What are all of the possible values of the mean of  $x$ ?