Sets

Math Lecture 1

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Set

A set is the most fundamental thing we will discuss.

We will use the intuitive definition of:
A collection of distinct elements.

Notation:

The set of elements containing elements a, b, c, and d is denoted

{a, b, c, d}

In a Set

a is in the set A is written as: $a \in A$

Examples of Sets

{a, b, c, d}

{Nick, Mike, Anne-Laure}

{4, 5, 6, …, 18}

{1, 3, 5, 7, 11, 13, …, 997}

{} **This is the set without any elements inside.**
It has a special name and symbol:
The empty set, $\emptyset$

Examples of Sets

$\mathbb{R}$ **The set of all real numbers**

$\mathbb{R}_+$ **The set of all positive real numbers**

$\mathbb{Q}$ **The set of all rational numbers**

$\{n \in \text{Prime} : n < 1000\}$

Order Doesn't Matter

Two sets with the same elements are the same set.

{a,b,c,d} = {c,b,d,a}

No Repetitions

Elements in a set cannot be repeated.

This is not a set: {a,a,b}

Set Operations

Union - yields a set with the elements of both sets

Example: {a,b,c,d} U {d,e,f,g} = {a,b,c,d,e,f,g}

Intersection - yields a set with just the elements that are common to both sets

Example: {a,b,c,d} ∩ {c,d,e,f} = {c,d}

Relationships

A is a subset of B means all the elements of A are in B.

$A \subset B$

A is a superset of B means all the elements of B are in A.

$A \supset B$

A is equal to B means that A is both a subset and a superset of B

A = B

Proving Sets are Equal

To prove two sets are equal, one must show:

All the elements in the first set are in the second set

All the elements in the second set are in the first set

Example: $A = \{1, 2, 3, 4\}$

$B = \{n \in \mathbb{Z}_+ : n^2 < 17\}$

Functions of Sets

Let S be a set of real numbers.

The following are functions sets of real numbers:

min, max, mean, median

Ordered Sets

An ordered set is a set where the order is specified.

Denoted as (a, b, c, d)

$$(a, b, c, d) \neq (c, a, d, b)$$

More Examples of Sets

$\mathbb{R}^2$ **All ordered sets of two elements of real numbers**

$$\mathbb{R}^2 = \{(a, b) : a \in \mathbb{R} \text{ and } b \in \mathbb{R}\}$$

$\mathbb{R}^3$ **All ordered sets of three elements of real numbers**

$$\mathbb{R}^3 = \{(a, b, c) : a \in \mathbb{R}, b \in \mathbb{R} \text{ and } c \in \mathbb{R}\}$$

$\mathbb{R}^n$ **All ordered sets of n elements of real numbers**

Vectors in $\mathbb{R}^n$

All ordered sets of n elements of real numbers together with pointwise addition and scalar multiplication

$$(a, b, c, d) + (e, f, g, h) = (a + e, b + f, c + g, d + h)$$

$$k(a, b, c, d) = (ka, kb, kc, kd)$$

Another Notation

Vectors in $\mathbb{R}^n$ can be written in a column.

Exs:

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \qquad a = \begin{bmatrix} 8 \\ \pi \\ e \end{bmatrix}$$